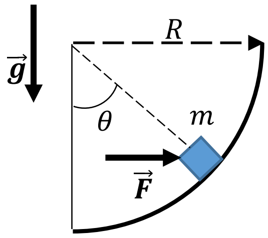
PHYS 101 Midterm Exam 2 Solution 2020-21-2

1. A block of mass m is moving up a frictionless circular track of radius R acted upon by a horizontal force $\vec{F}$ as shown in the figure. Magnitude F of the force is adjusted so that the block moves up the circular track with constant speed v.



(a) (5 Pts.) Draw a free-body diagram of the block.

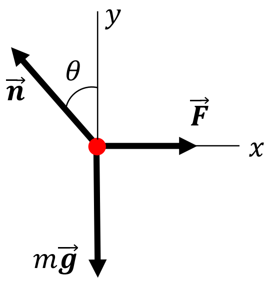
(b) (5 Pts.) What is the magnitude F of the applied force as a function of θ which causes the block move up the circular track with constant speed?

(c) (5 Pts.) What is the work done by the gravitational force as the block moves from $\theta = 0$ to $\theta = \theta_f$ ($\theta_f < \pi/2$)?

(d) (5 Pts.) What is the work done by the force $\vec{F}$ as the block moves from $\theta = 0$ to $\theta = \theta_f$?

(e) (5 Pts.) What is the average power delivered to the block by the force $\vec{F}$ as it moves from $\theta = 0$ to $\theta = \theta_f$?

Solution: (a)



(b) Writing Newton's second law, we have

$$F - n\sin\theta = -ma_{\text{rad}}\sin\theta\,,\, n\cos\theta - mg = ma_{\text{rad}}\cos\theta \quad \rightarrow \quad F = mg\tan\theta\,.$$

(c) Work done by the gravitational force as it moves from $\theta = 0$ to $\theta = \theta_f$ is related to the change in the gravitational potential energy of the block. Taking the initial position $\theta = 0$ as the zero level for the potential energy, we have

$$W_g = -\Delta U = -(U_f - U_i) = -mgR(1 - \cos\theta_f)\,.$$

(d) Since the speed of the block is constant, its kinetic energy does not change. According the the work-energy theorem, this means that the net work done on the block is zero.

$$W_F + W_g + W_n = 0\,.$$

The work done by the normal force is zero. This means

$$W_F + W_g = 0 \quad \rightarrow \quad W_F = -W_g \quad \rightarrow \quad W_F = mgR(1 - \cos\theta_f)\,.$$

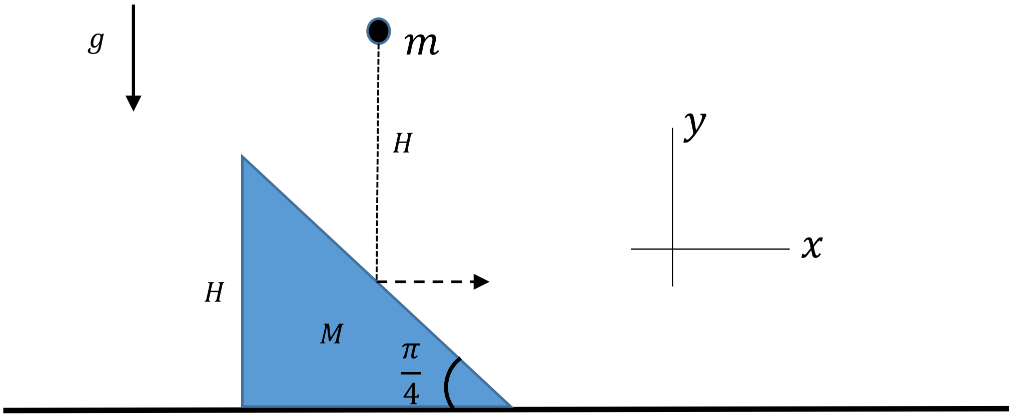
(e) The block coveres a total distance $R\theta_f$ moving with speed v means that the time it takes is $T = R\theta_f/v$. Therefore, average power delivered to the block by the force $\vec{F}$ as it moves from $\theta = 0$ to $\theta = \theta_f$ is

$$P_{\text{av}} = \frac{W_F}{T} \quad \rightarrow \quad P_{\text{av}} = \frac{mgRv}{R\theta_f}(1 - \cos\theta_f)\,.$$

2. A right-angle inclined plane of mass M, side H, and angle $\pi/4$ can slide on a horizontal floor without friction. Initially, the inclined plane is at rest. A ball of mass m is dropped onto the inclined plane's midpoint from a height H, as shown in the figure. The collision is elastic, and the ball's velocity is horizontal right after the collision. Gravitational acceleration is g.

a) (15 Pts.) Find the velocity of the inclined plane after the collision.

b) (10 Pts.) What is the distance between the ball and the right edge of the inclined plane when the ball hits the floor?



Solution: (a) Choosing the initial position of the ball as zero gravitational potential energy level, and using the fact that total mechanical energy is conserved during the fall, the downward speed of the ball just before it collides with the inclined plane is found as

$$\Delta E = \frac{1}{2}mv^2 - mgH = 0 \quad \rightarrow \quad v = \sqrt{2gH}.$$

Since the ball collides elastically with the inclined plane, kinetic energy of the system is conserved. Also, since there is no external force in the *x*-direction, the *x*-component of the total momentum is conserved. These mean

$$\frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 = mgH\,, \qquad mv_1 + Mv_2 = 0 \quad \rightarrow \quad v_1 = \sqrt{\frac{2MgH}{M+m}}\,, \qquad v_2 = -\frac{m}{M}\sqrt{\frac{2MgH}{M+m}}.$$

(b) The motion of the ball after the collision will be a projectile motion with a horizontal initial velocity v_1 starting from a height $H/2$ above the floor. Therefore, it will hit the floor when

$$y_1 = \frac{H}{2} - \frac{1}{2}gt^2 = 0 \quad \rightarrow \quad t = \sqrt{\frac{H}{g}}.$$

During this time the ball will be displaced by an amount $\Delta x_1 = v_1 t = \sqrt{\frac{2M}{M+m}}H$ (to the right), while the inclined plane will be displaced by $\Delta x_2 = v_2 t = -\frac{m}{M}\sqrt{\frac{2M}{M+m}}H$ (to the left). Therefore, the distance D between the ball and the right edge of the inclined plane when the ball hits the floor will be $D = \Delta x_1 - \Delta x_2 - H/2$, or

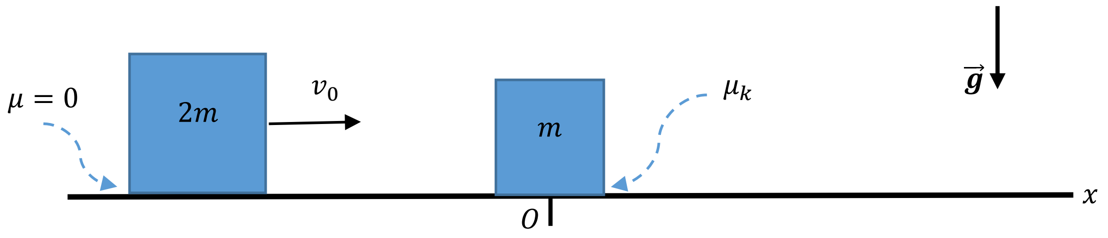
$$D = \left[\sqrt{2\left(1+\frac{m}{M}\right)} - \frac{1}{2}\right]H.$$

3. Consider a block of mass $2m$ moving on a horizontal surface with speed v_0 towards another block of mass m, which is initially at rest at $x = 0$. The $2m$ block can move without friction, while the kinetic friction coefficient between the horizontal surface and the other block is μ_k. Gravitational acceleration is g.

If all the collisions between the two blocks are elastic:

a) Find the time interval between the first and the second collision.

b) Find the final positions and velocities of both blocks.



Solution:

(a) Total momentum in the x-direction is conserved in collisions. Therefore, after the first collison, we have

$p_{xi} = 2mv_0\,, \qquad p_{xf} = 2mv'_1 + mv'_2\,, \qquad p_{xi} = p_{xf} \quad \rightarrow \quad 2v'_1 + v'_2 = 2v_0.$

Also, since all collisions are elastic, kinetic energy of the system is also conserved. This means

$v_1 - v_2 = -(v'_1 - v'_2) \quad \rightarrow \quad -v'_1 + v'_2 = v_0.$

Solving this set of two equations for the two unknowns, we find velocities of the two blocks after the first collision as

$v'_1 = v_0\,/3\,, \qquad v'_2 = 4\,v_0\,/3.$

Following the collison, the $2m$ block will move with constant velocity so its position is $x_1 = \frac{1}{3}v_0 t$. But, the block with mass m will slow down because of friction until it stops at $t_f = \frac{4\,v_0}{3\,\mu_k g}$. Its position is

$$x_2 = \begin{cases} \dfrac{4}{3}v_0 t - \dfrac{1}{2}\mu_k g t^2, & t < t_f \\ \dfrac{8\,v_0^2}{9\,\mu_k g}, & t \geq t_f \end{cases}.$$

Therefore, the two blocks will collide again when

$$x_1 = x_2 \quad \rightarrow \quad t = \frac{8v_0}{3\mu_k g}.$$

(b) Because all collisions are elastic, the system will stop when the work done by the friction is equal to the initial kinetic enegy of the system.

$\Delta E = W_f \quad \rightarrow \quad -\frac{1}{2}(2m)v_0^2 = -\mu_k mgd \quad \rightarrow \quad d = \frac{v_0^2}{\mu_k g}\,, \; v_{1f} = v_{2f} = 0.$ This result can also be obtained alternatively by summing the infinite series of displacements after each collision:

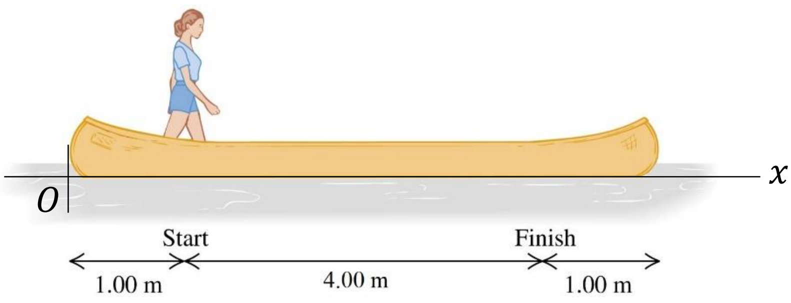
$$d = \frac{8\,v_0^2}{9\,\mu_k g}\left(1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^4 + \cdots\right) = \frac{v_0^2}{\mu_k g}.$$

4. A 40.0-kg woman stands up in a 60.0-kg canoe 6.00 m long which is initially at rest floating on a smooth lake. She walks from a point 1.00 m from one end to a point 1.00 m from the other end in 4 seconds. Assume that the canoe has a uniform mass distribution, ignore resistance to motion of the canoe in the water, and use the coordinate system shown in the figure so that initially the woman is at $x = 1.00$ m.

(a) (6 Pts.) What is the *x*-coordinate of the center of mass of the woman-canoe system before the woman starts to walk?

(b) (6 Pts.) What is the velocity of the canoe with respect to the lake surface as the woman is walking.

(c) (6 Pts.) What is the velocity of the canoe after the woman stops at the other end?



(d) (7 Pts.) How far does the canoe move during this process?

Solution: (a)

$$x_{\text{CM}} = \frac{M_{\text{W}}x_{\text{W}} + M_{\text{C}}x_{\text{C}}}{M_{\text{W}} + M_{\text{C}}} = \frac{(40.0\text{ kg})(1.0\text{ m}) + (60.0\text{ kg})(3.0\text{ m})}{100\text{ kg}} \quad \rightarrow \quad x_{\text{CM}} = 2.2\text{ m}$$

(b) The sum of all external forces acting on the woman-canoe system is zero. When the woman starts walking towards the other end of the canoe, the canoe will move in the opposite direction to conserve momentum. The speed of the woman with respect to the canoe is

$$v_{W/C} = (4.0\text{ m})/(4\text{ s}) = 1.0\ \text{m/s}\,.$$

$$v_{W/L} = v_{W/C} + v_{C/L} \quad \rightarrow \quad v_{W/L} = (1.0\ \text{m/s}) + v_{C/L}.$$

Since momentum in the *x*-direction is conserved, we have

$$M_{\text{W}}v_{W/L} + M_{\text{C}}v_{C/L} = 0 \quad \rightarrow \quad M_{\text{W}}\left(1 + v_{C/L}\right) + M_{\text{C}}v_{C/L} = 0 \quad \rightarrow \quad v_{C/L} = -\frac{M_{\text{W}}}{M_{\text{W}} + M_{\text{C}}}(1.0\ \text{m/s})$$
$$= 0.4\ \text{m/s}\,.$$

(c) Zero, because of momentum conservation.

(d) The position of the center of mass of the woman-canoe system does not change during the process. Therefore, asuming that the canoe moves a distance d to the left, we have

$$\frac{M_{\text{W}}[(5\text{ m}) - d] + M_{\text{C}}[(3\text{ m}) - d]}{M_{\text{W}} + M_{\text{C}}} = 2.2\text{ m} \quad \rightarrow \quad d = 1.6\text{ m}\,.$$

Same answer can also be found as $d = v_{C/L}\,T = (0.4\ \text{m/s})(4\text{ s}) = 1.6\text{ m}\,.$